Robust Credit Stress Testing Through a Cointegrated Framework

Tiziano Bellini

Abstract Stress testing has become an important topic in the banking sector since the development of the risk management and the enforcement of banking international supervisory requirements. International standards, however, do not define how to stress risk parameters. In our research we extend the forward search (FS) to cointegration showing the FS effectiveness in detecting atypical units. Analyzing Italian time series, we build up a framework to emphasize the interaction among macroeconomic variables and their impact on banking credit risk parameters.

Key words: Stress testing, cointegrated vector autoregressive (CVAR) model, forward search (FS).

1 Introduction

In the last few years, stress testing has become a crucial theme in the financial literature. This topic has been explored focusing essentially on the impact of macroeconomic shocks on the financial system as a whole. More recently this process has been investigated from a different point of view. From the risk management perspective, the European Banking Authority (EBA, 2011) defines a stress testing framework to be exploited to assess banking capital when extreme events take place. This stress testing process relies on the shock of credit risk variables such as the probability of default (*Pd*), the loss given default (*Lgd*) and so on. From a practical point of view, however, regulators do not specify how to stress the above mentioned variables. Thus, we aim to build up a coherent framework to robustly estimate a macroeconomic model to be applied for stress testing purposes. We estimate a cointegrated vector autoregressive (CVAR) model using the PML approach, proposed by Franses

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and Lucas (1998), and the FS extended to the CVAR model by Bellini (2013). Applying our analysis on Italian data from 1990 to 2009, we can summarize our main contributions as follows:

- Robust estimation of CVAR model parameters.
- Analysis of the interaction between credit risks and macro-economic variables. Identification of the long run stressed macro-economic equilibrium to shock credit risk parameters.

In Section 2 we concentrate on the model to link credit risk variables to macro-economic factors. In Section 3 we apply our framework to real time series. Section 4 contains concluding remarks and directions for future research.

2 Cointegrated Stress Testing Framework

According to the Basel capital requirement formula, we stress credit risk variables considering the asymptotic risk factor (ASRF) default model (Gordy, 2003). In this context, default events are modeled through a random variable ξ assuming value 1 in the case of default and 0 otherwise. In a portfolio of n debtors, we consider as key risk parameter to stress the Pd_j, $(j = 1, \ldots, n)$, i.e., $P[\xi_j = 1]$. In the ASRF approach, we can represent Pd_j conditioned on a given state of the economy X_1^* as follows

$$P[\xi_j = 1 | X_1 = X_1^*] = \Phi\left[\frac{\Phi^{-1}(Pd_j) - \rho_j X_1^*}{\sqrt{1-\rho_j}}\right], \tag{1}$$

where X_1 is an index representing the state of the economy, Φ is the standard Normal cdf and ρ_j represents the correlation between debtor j and the economy. In our framework, given a macro-economic scenario, we are interested in stressing X_1 and compute the impact of this shock on Pd_j, $(j = 1, \ldots, n)$, through equation (1). According to the macro-economic literature, we can exploit the CVAR model in the VECM form to investigate X_1 together with a set of macro-economic variables X_i, $(i = 2, \ldots, p)$. In this transmission mechanics, we are interested in studying what happens when applying a shock to one or (jointly) many macro-economic variables X_i, $(i = 2, \ldots, p)$ on X_1 and, therefore, on Pd_j. The CVAR model in the VECM form can be represented as follows

$$\Delta X_t = \Pi X_{t-1} + \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_{k-1} \Delta X_{t-k+1} + \Xi D_t + \varepsilon_t, \tag{2}$$

where X_t, $(t = 1, \ldots, T)$, is a p-dimensional vector, ΔX_{t-l}, $(l = 1, \ldots, k-1)$, represents the first difference computed over $(t-l)$ and $D_t = (D_{1,t}, \ldots, D_{g,t}, \mu_0)$ contains g dummies and a constant. The matrix Π can be represented as the matrix product $\alpha\beta'$. It is assumed that errors are Normally distributed $\varepsilon_t \sim N(0, \Omega)$.

We can summarize the framework as follows:

- The goal of the analysis is to compute stressed default probabilities, $Pd_{j,stress}$, shocking macro-economic variables.

- Equation (1) describes the functional relationship between Pd_j and X_1 (index representing the state of the economy).
- Starting from parameter estimates of equation (2), we shock macro-economic variables ΔX_i, $(i = 2, \ldots, p)$, to obtain the stressed state of the economy, $\Delta X_{1,stress}$, and compute stressed default probabilities $Pd_{j,stress}$.

In order to estimate equation (2), residuals play a key role. In addition, they are at the core of both the pseudo-maximum-likelihood (PML) approach as well as the forward search (FS) analysis. In particular, the search moves forward considering the subset $S_*^{(m)}$ consisting of the observations with the m smallest standardized residuals $r_t(m^*)$ (Bellini, 2013). We indicate the observation with the minimum residual among those $\notin S_*^{(m)}$ as follows

$$t_{min} = \arg\min[r_t^*(m^*)] \quad t \notin S_*^{(m)}. \tag{3}$$

To test whether observation t_{min} is an outlier, we rely on the following statistics

$$r_{tmin}^*(m^*) = \sqrt{\frac{e_{tmin}(m^*)' e_{tmin}(m^*)}{s^2(m^*)}}. \tag{4}$$

In the next section we estimate equation (2) parameters on Italian real time series.

3 Credit Stress Testing in Practice

Our analysis relies on Bank of Italy, ISTAT and other financial quarterly data from 1990 to 2009. In particular, our dataset X_t is made up by the variable $X_{1,t}$, which measures the health of the economy through default rates (DR) computed by the Bank of Italy on the whole Italian economy, and $(p-1)$ macro-economic variables. The set of macro-economic variables is constituted by: gross domestic product (GDP), unemployment rate (UR), real estate index (RE), Euro-Dollar exchange rate (ER), Euribor 3 months interest rate (EUR3) and Italian 10 years treasury bond's interest rate (BTP10). The choice of these variables, is strictly connected with the EBA (2011) stress testing framework.

Remarking that the cointegration analysis is particularly sensitive to outlying observations, we exploit a robust approach to carry out the study. The first step is to estimate the CVAR model and verify whether assumption underlying equation (2) hold. Aiming to robustly estimate the CVAR model, we introduce the following analysis carried out relying on the PML and FS analysis implemented in Matlab R2011. In Figure 1, panel (a) shows re-scaled X_t time series from 1990 to 2009 (72 observations). In order to check for the presence of outliers, in panel (b) we draw PML weights. We concentrate on $\nu = 5$, showing the related critical value. This panel shows that observations: 18 (1997:02), 19 (1997:03), 64 (2008:04) and 65 (2009:01) cross or are very close to the critical value. They can be considered outlying units. Moving to the FS analysis, the bottom panels of Figure 1 show the

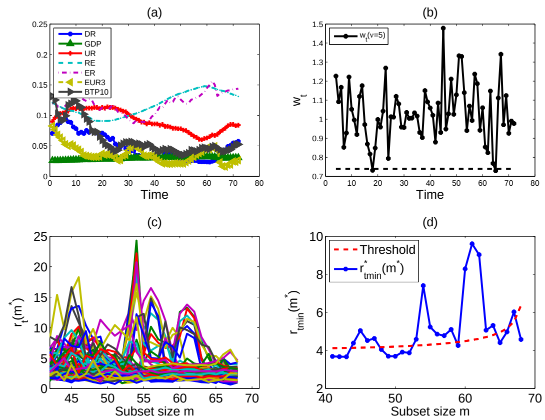

Fig. 1 CVAR time series analysis: PML vs. FS approach.

evolution of residuals along the search. Panel (c) shows the evolution of standardized residuals $r_t(m^*)$ for each observation. Each line of this plot shows $r_t(m^*)$, for unit t, $(t = 1,\ldots,T)$, as the subset size m increases. We remind that $r_t(m^*)$ stands for standardized residual for unit t at step m, obtained considering the parameter vector $\hat{\Theta}^{(m)}_*$. In Figure 1, panel (c) shows that there are some lines very close each others with small $r_t(m^*)$, while there are some other trajectories with higher $r_t(m^*)$. We are induced to think about the presence of a cluster of outliers. However, at the end of the search, there is an evident reduction of $r_t(m^*)$ for units belonging the above mentioned cluster. The shape of these trajectories shows an evident masking effect. In order to make inference on outliers, in panel (d), we compare $r^*_{tmin}(m^*)$ to the 99% confidence threshold. From this panel, we notice that $r^*_{tmin}(m^*)$ curve crosses the 99% threshold at different subset sizes. According to what we highlighted in panel (c), the masking effect is confirmed by the fact that in the last few steps of the search $r^*_{tmin}(m^*)$ is below the threshold. The crucial point is step 62 where a big jump shows the presence of a cluster of atypical units.

Aiming to figure out how the FS helps the researcher to carry out the study, we introduce some changes in our original model. In particular, we consider a structural break from unit 10 (1995:02) to 16 (1996:04) corresponding to outlying units at step 62. In addition, according to the PML approach and in accordance to the last few steps of the FS, we consider dummy variables corresponding to units 64 (2008:04) and 65 (2009:01). We run this model in CATS obtaining the misspecification test outputs summarized in Table 1. Distinguishing between the original and the model with dummy, Table 1 shows that the model specification improves deeply. In particular, in the robust model ARCH and Normality cannot be rejected while they are in the original model. In Table 2 we report CVAR rank tests. From this analysis, in the original model it is not clear whether to conduct the analysis considering $r = 3$ or $r = 4$. In the robust model, the trace test suggests $r = 4$ ($p-value$=0.223).

	Original Model	Robust Model
Autocorr.$\chi^2(49)$,$[p-value]$	72.034 [0.018]	62.929 [0.087]
Norm.$\chi^2(14)$, $[p-value]$	23.503 [0.053]	13.994 [0.450]
ARCH $\chi^2(1568)$,$[p-value]$	1655.197 [0.062]	1627.057 [0.146]
Trace corr.	0.537	0.633

Table 1 Multivariate misspecification tests: unrestricted model.

		Original Model			Robust Model		
p-r	r	Eig.Value	Trace	p-value	Eig.Value	Trace	$p-value$
7	0	0.635	217.170	[0.000]	0.79	310.604	[0.000]
6	1	0.493	146.688	[0.000]	0.607	201.243	[0.000]
5	2	0.424	99.200	[0.006]	0.495	135.826	[0.005]
4	3	0.267	60.620	[0.089]	0.39	88.049	[0.068]
3	4	0.203	38.894	[0.119]	0.313	53.452	[0.223]
2	5	0.184	23.050	[0.108]	0.211	27.211	[0.440]
1	6	0.119	8.833	[0.196]	0.141	10.666	[0.474]

Table 2 Trace test of the cointegragion rank: unrestricted model.

We are now interested in testing whether β vector is stationary. It is evident from Table 3 that all the variables are stationary. When focusing on the vector α, on the one hand, we notice that GDP, ER and EUR3 are weakly exogenous, i.e. they can be regarded as non-equilibrium correcting. On the other hand, the unit vector test emphasizes that GDP and RE are purely adjusting, they do not contribute to common trends. Therefore, shocks to such variables have no permanent effect on any of the variables in the system. Thus we can emphasize that the FS analysis allows improving model estimation in terms of specification testing and fitting. The break detected through the FS is not shown through the PML approach. In addition, the key benefit of using the FS is that the entire CVAR analysis can be carried out exploiting the well-known CVAR literature. The identification process as well as the CVAR structural analysis can be accomplished exploiting usual tools without having to face new theoretical challenges due to weighting schemes adopted in other robust techniques such as the PML approach.

	DR	GDP	UR	RE	ER	EUR3	BTP10
Tests on β							
Stationarity, $\chi^2(p-r)$	5.583	0.585	1.012	1.619	2.198	1.214	1.879
$[p-value]$	[0.134]	[0.900]	[0.798]	[0.655]	[0.532]	[0.750]	[0.598]
Tests on α							
Weak exogeneity, $\chi^2(r)$	20.744	7.738	10.913	26.788	5.644	3.698	50.747
$[p-value]$	[0.000]	[0.102]	[0.028]	[0.000]	[0.227]	[0.448]	[0.000]
Unit vector, $\chi^2(p-r)$	10.127	1.749	12.018	1.881	13.848	14.951	7.978
$[p-value]$	[0.018]	[0.626]	[0.007]	[0.597]	[0.003]	[0.002]	[0.046]

Table 3 Hypothesis tests on β and α for the rank $r = 4$.

The last step of the analysis is the estimation of the long run macro-economic equilibrium. The moving average (MA) or Granger-Johansen Johansen (1996) representation of the CVAR is given by

$$X_t = C\sum_{j=1}^{t}(\varepsilon_j + \phi D_j) + C^*(L)(\varepsilon_t + \phi D_t) + X_0, \tag{5}$$

where $C = \beta_\perp(\alpha'_\perp \Gamma \beta_\perp)^{-1}\alpha'_\perp \equiv \tilde{\beta}_\perp \alpha'_\perp$ with $\Gamma = I - \Gamma_1 - \ldots - \Gamma_{k-1}$. $\alpha'_\perp \sum_{j=1}^{t} \varepsilon_j$ defines the $p-r$ non-stationary common trends loaded by coefficients in $\tilde{\beta}_\perp$, while $C^*(L)\varepsilon_t$ denotes the stationary part of the process. X_0 is a function of initial conditions. We estimate $\hat{C}$ starting from our identified model. Thus, according to what we stated above we are able to stress Pd_j through equation (1).

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4 Concluding Remarks

We propose a CVAR framework to stress credit risk variables starting from shocked macro-economic variables. This framework can be used first of all to define a coherent stressed scenarios starting from the shock of one or different macro-economic variables in order to estimate the impact on the index representing the health of the economy. At the same time, the proposed framework can be exploited to estimate the impact on this index starting from exogenous scenarios such as EBA (2011). This is a first step in applying a robust CVAR approach for stress testing purposes, further studies need to be devoted to extend this approach to integrate different risks.

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